J. Fluid Mech. (1967), *vol.* 29, *part* 2, *pp.* 233–240
Printed in Great Britain

The stability of a stratified fluid

By J. B. HINWOOD
Department of Mechanical Engineering, Monash University, Melbourne

(Received 22 August 1966 and in revised form 15 January 1967)

For the flow of a stably-stratified fluid in the inlet region of a rectangular duct, it is shown experimentally that the upper and lower critical Reynolds numbers are functions of the interfacial Froude number F, and that if F is large they are lower than for a homogeneous flow. In stratified flows the disturbances leading to turbulent flow sometimes arise at the interface and lead to interfacial waves, whose wavelength at breaking is equal to the conduit depth.

1. Introduction

The stability of the interface between two liquid strata of equal depth and discharge has been investigated for the case of no relative motion between the strata. Hence the presence of viscous shear at the interface is no longer the primary cause of instability. This situation has also been studied recently by Charles & Lilleleht (1965) but they used two immiscible strata with a relatively large difference in density, whereas the present study is concerned with two miscible strata with a very small difference in density.

The experimental investigations described in this paper may be divided into three almost independent parts. The first of these parts, of which a preliminary report has already been given (Macagno & Hinwood 1964), is concerned with demonstrating that a two-layered flow with no interfacial shear may be less stable than a homogeneous flow, and with classifying and establishing the conditions of occurrence of the different flow regimes. In the second part of the investigation, the disturbances which lead to instability are examined. These disturbances are interfacial waves, whose wavelength is almost constant over the range of conditions studied. These two parts of the study did not reveal the mechanisms through which the unstable disturbances originated and grew so flow-visualization techniques were used to show where the disturbances first appeared and to follow their development.

2. Experimental apparatus

The experimental apparatus, shown in figure 1, consisted of a rectangular conduit of black lucite, 6 in. wide, 2 in. deep and 130 in. long, connected to a large stationary head tank at the upstream end, and a flow meter and a small moveable overflow tank at the downstream end. The latter tank was driven by a variable-speed motor at such a speed that it maintained the desired constant discharge through the unit.

The head tank had an internal partition reaching from the top down to a level below the streamlined inlet to the conduit. This tank was initially two-thirds filled with salt water, then fresh water was carefully floated on top of the salt water in the section of the tank containing the conduit inlet until the interface was depressed to the level of the centre line of this inlet. Any mixed interlayer was then withdrawn through the conduit. By suitably proportioning the cross-sectional areas of the two parts of the tank, equal discharges of fresh and salt water could be obtained, without any external adjustments being required during the course of an experiment.

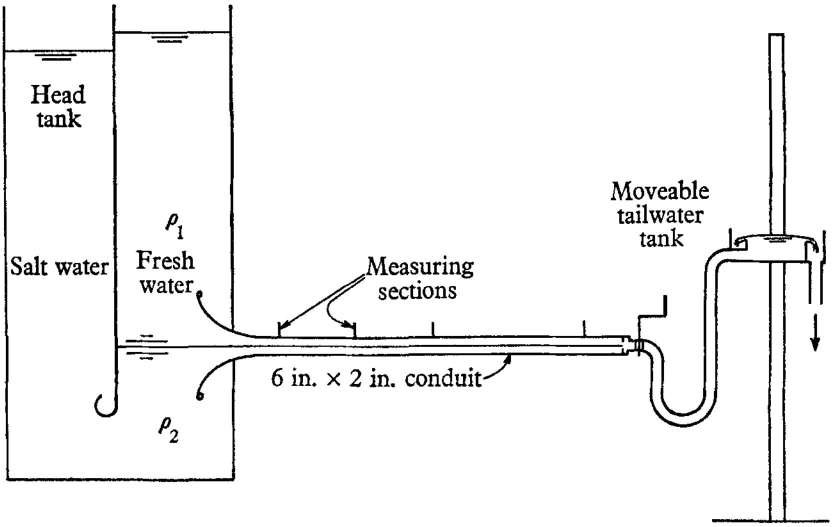


FIGURE 1. Experimental apparatus.

With this apparatus, steady two-layer flows could be produced and maintained for about 25 min. at the maximum discharge used. The velocity distribution at the inlet was very nearly uniform, as desired, but the conduit was not long enough to obtain fully developed flow at the downstream end, however, this did not prove to be a disadvantage. The most serious shortcoming of the unit was its breadth to depth ratio of three, which was too small to give two-dimensional flow, which led to the centre line velocity being greater than it would have been in a two-dimensional flow. Another shortcoming was the fact that preparation for each experimental run took about 4 days, which imposed a limit on the number of experiments which could be performed.

The conduit was fitted with four windows through which detailed visual observations of the flow were made, and velocity profiles were obtained by photographing neutrally buoyant droplets illuminated by a stroboscope. Density profiles were obtained at the same sections, at first by withdrawing very small samples of water through a rake of hypodermic needles and subsequently measuring the resistance of the samples using a conductivity cell, and later using a traversing conductivity probe to directly plot the density profiles. The probe, which is described elsewhere (Macagno & Hinwood 1965) consisted of two concentric electrodes, the outer one being the end of a hypodermic needle.

Both the conductivity cell and the probes were calibrated for each experiment by gravimetrically evaluating the density of a number of reference solutions. The output voltage of the probe circuit varied linearly with density over the range $0{\cdot}0012 < \Delta\rho/\rho < 0{\cdot}040$ used in these experiments.

The flow was not deliberately disturbed and as the natural frequencies of structural oscillations of the apparatus, as measured by an accelerometer, were much higher and the frequency of the pendulation in the head tank was much lower than the observed interfacial disturbance frequency of about 2 c/s, the disturbances which did appear were not forced oscillations. The naturally present perturbations were sufficient to cause the breakdown of an unstable flow with acceptable reproduceability.

3. Experimental results

Conditions for stability

As a measure of the amount of interfacial mixing, a non-dimensional transport parameter T may be defined as the sum of the amounts of both fluids mixed across the interface during the period of travel from the inlet to the given section distant x from the inlet. T is defined by the expression

$$T = \frac{\int_h^{2h} (\rho - \rho_1) u\, dy + \int_0^h (\rho_2 - \rho) u\, dy}{Uh\,\Delta\rho},$$

where the subscripts 1 and 2 refer to the upper and lower strata respectively, $\Delta\rho = \rho_2 - \rho_1$, ρ and u are respectively the density and the horizontal component of velocity at any point (x, y) and U is the mean velocity. Since $\Delta\rho/\rho$ is very small, the thicknesses of the strata have been taken as equal to h, the half-depth of the conduit.

With this approximation, and subsequently neglecting the inertial effects of the stratification, we may write

$$\begin{aligned} T &= fn(\rho_1, \rho_2, \mu, x, h, g, U_c) \\ &= fn(R, F, x/h), \end{aligned}$$

where the interfacial Froude number is $F = U_c/(gh\Delta\rho/\rho)^{\frac{1}{2}}$, the Reynolds number is $R = U_c h/\nu$, ν is the mean kinematic viscosity and U_c is the centre line velocity. An alternative formulation in terms of the Reynolds numbers of each layer, used by Charles & Lilleleht, is unsuitable for this situation as it would conceal the influence of gravitational forces and would wrongly suggest that second-order differences in viscous and inertial forces between the two strata were responsible for instability. No mixing of the strata means that $T = 0$ and corresponds to the stable and neutral portions of the parameter space, other surfaces of constant T define boundaries between zones of different stability behaviour.

The values of T from the four measuring sections have been plotted in figure 2 as a function of R and a combination of F and x/h. No analytical basis for the combination of F and x/h could be found, and the scatter of the data makes it difficult to give strong support to one combination in preference to another. Using this

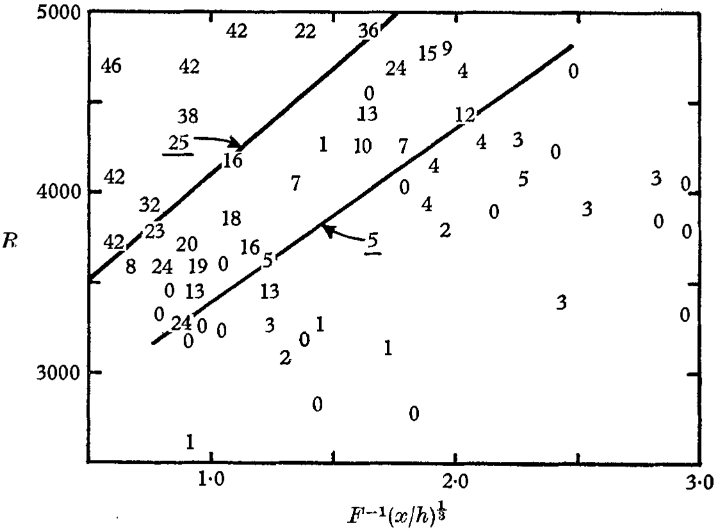
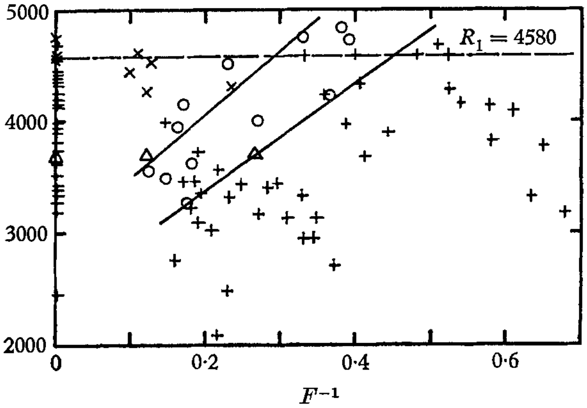
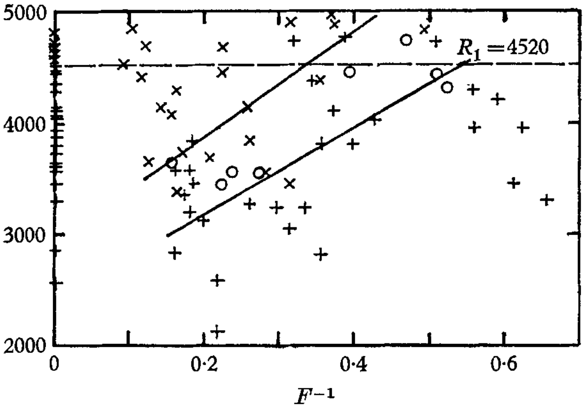


FIGURE 2. Classification of flow regimes based on mixing. Values of $T \times 10^3$.



(*a*) $x/h = 41{\cdot}4$



(*b*) $x/h = 65{\cdot}4$

FIGURE 3. Visual classification of flow regimes. +, stable; ○, transition; ×, unstable.

figure the two transverse lines separating the three regions of the flow were obtained. The value of T at the lower limit of transition was chosen just larger than zero to compensate for the small spatial averaging involved in the density measurement. The mean value of T increases in the transition region where there appears to be more mixing than actually occurred, because of the presence of interfacial waves. Intense mixing actually occurred in the unstable region and led to the large values of T shown in the figure. The lines obtained from figure 2 have been plotted in figures 3(*a*) and 3(*b*), where it may be seen that they agree with the visual classification of the flows as stable, transitional or unstable. In each of these figures a broken horizontal line has been drawn at the transition Reynolds number, $R = R_1$, for the flow of a homogeneous liquid in this unit.

The stratified flows showed instability at lower Reynolds numbers than R_1 which indicates that they are sometimes less stable than homogeneous flows. That their modes of instability may also be different was shown by the visual observations as is explained below. In all cases the flow was stable at the inlet of the conduit.

Each of the classes, stable, transition and unstable, was actually made up of several sub-classes. The finer sub-division showed that while homogeneous flows passed abruptly from stable to unstable flow, the stratified flows usually passed through the following stages (figure 6, plate 1):

(i) laminar flow with straight parallel streamlines;
(ii) some streamlines slightly curved;
(iii) interface slightly curved;
(iv) small interfacial waves present, but not breaking;
(v) interfacial waves present, some breaking, remainder of flow laminar;
(vi) larger interfacial waves, some breaking, little mixing;
(vii) turbulent zone around the interface due to breaking waves.

Stage (iii) marks the lower limit of transition and the upper limit lies between stages (iv) and (vi). The waves were usually three-dimensional and were not strongly periodic.

Interfacial waves were observed visually in the course of the above experiments but more exact measurements were desired. For this reason the conductivity probes which had been developed for density-profile measurement were adapted to permit the level of the interface to be recorded as a function of time. This was achieved by increasing the separation of the electrodes and hence enlarging the volume of fluid carrying the electric current to such an extent that a probe positioned at the mean level of the interface could measure interfacial waves of amplitudes up to 0·1 in.

To produce the disturbances which lead to instability, the discharge was increased until occasional interfacial waves broke. The fluctuations in the level of the interface were then recorded. This was done for one value of the density difference, at each of the three upstream windows, for flows spanning the whole transition range, with a few runs in the stable and unstable regions.

Typical profiles of interfacial waves are shown in figure 4. It is apparent that when they first appear the waves are almost periodic—a typical log-normal frequency distribution is given in figure 5. Figure 4 also shows that the wave amplitude and frequency increase with distance from the conduit inlet and with Reynolds number. From these measurements it was found that the dominant frequency was inversely proportional to U_c, and hence the wave length λ was constant, see figure 8. It was found that $2{\cdot}0h < \lambda < 2{\cdot}8h$. This result was confirmed by the visual observations made during the preceding series of experiments, where it was found that in 85 % of all cases $2h < \lambda < 2{\cdot}5h$. Hence the tentative conclusion is reached that $\lambda \simeq 2{\cdot}4h$ for incipiently unstable interfacial waves. An important restriction is that h was not varied in these experiments.

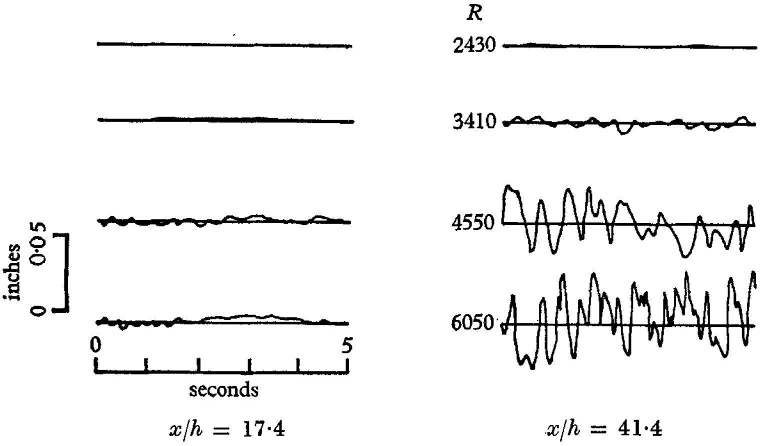


FIGURE 4. Interfacial wave records. $\Delta\rho/\rho = 0{\cdot}00145$.

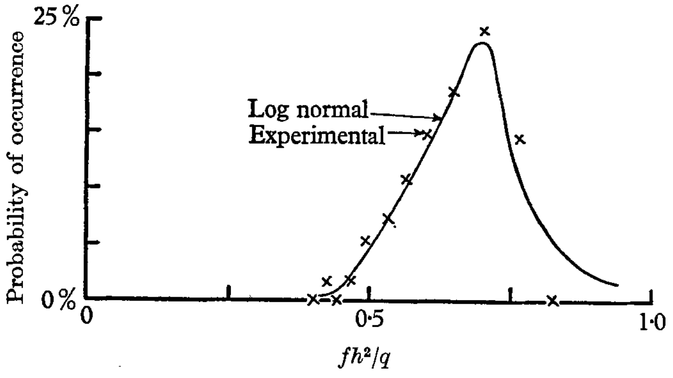


FIGURE 5. Frequency distribution of interfacial waves. $R = 3410$. $F^{-1} = 0{\cdot}097$; based on 54 samples each of 4 wavelengths.

Origin and growth of disturbances

The measurements of mixing, the visual observations and the interfacial wave records failed to reveal the origin of the disturbances, and so various flow visualisation techniques were tried.

The most successful technique was the addition of shampoo which by refraction effects revealed the presence of disturbances, but because shampoo solution is slightly non-Newtonian the following results are only descriptive. The transition from stable to unstable flow is different in stratified and homogeneous flows as is shown in figure 6, plate 1 and in figure 7, plate 2, where the shampoo acted like a dye to reveal the growth of an unstable wave which originated at the interface. Dye photographs showed that such waves frequently did not disturb the wall boundary layer which remained steady and laminar. The shampoo technique also revealed disturbances which originated in the wall boundary layer and produced interfacial waves.

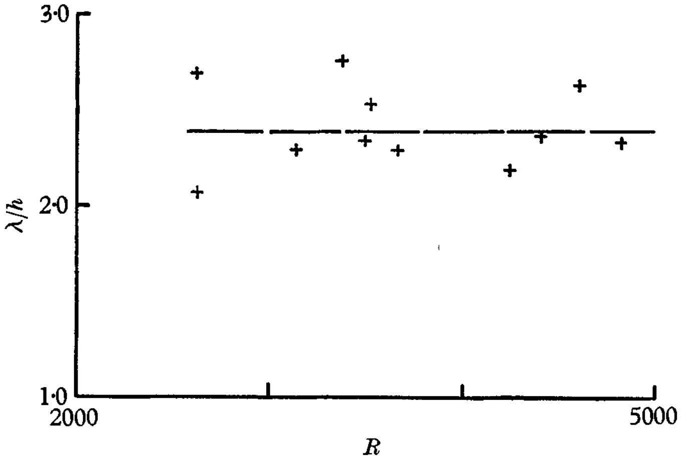


FIGURE 8. Wavelengths of interfacial waves near breaking. $\Delta\rho/\rho = 0{\cdot}00145$.

In such cases an eddy was produced in the boundary layer, the interface was sucked in behind it and then returned abruptly to its normal level, forming a sharply peaked wave which tended to grow and ultimately to break. There is no evidence of a resonant type of energy transfer to interfacial waves from Tollmien–Schlichting waves in the boundary layer, and indeed the frequencies of the two types of waves were quite different.

4. Conclusions

The experimental investigations showed:

(i) that a stably-stratified flow with no interfacial shear is sometimes less stable than an unstratified flow, despite the stabilizing influence of gravitational forces;

(ii) that its mode of transition is different;

(iii) that the point of transition depends on the interfacial Froude number, as well as the Reynolds number and the distance from the inlet;

(iv) that the stratified flows are subject to disturbances arising near the interface as well as in the boundary layer;

(v) that the intensity of these disturbances increases with the Reynolds number; and

(vi) that interfacial disturbances need not disturb the boundary layer.

Several flows of this type have been simulated numerically, and the solutions which are to be presented in a later paper agree with the experimental results.

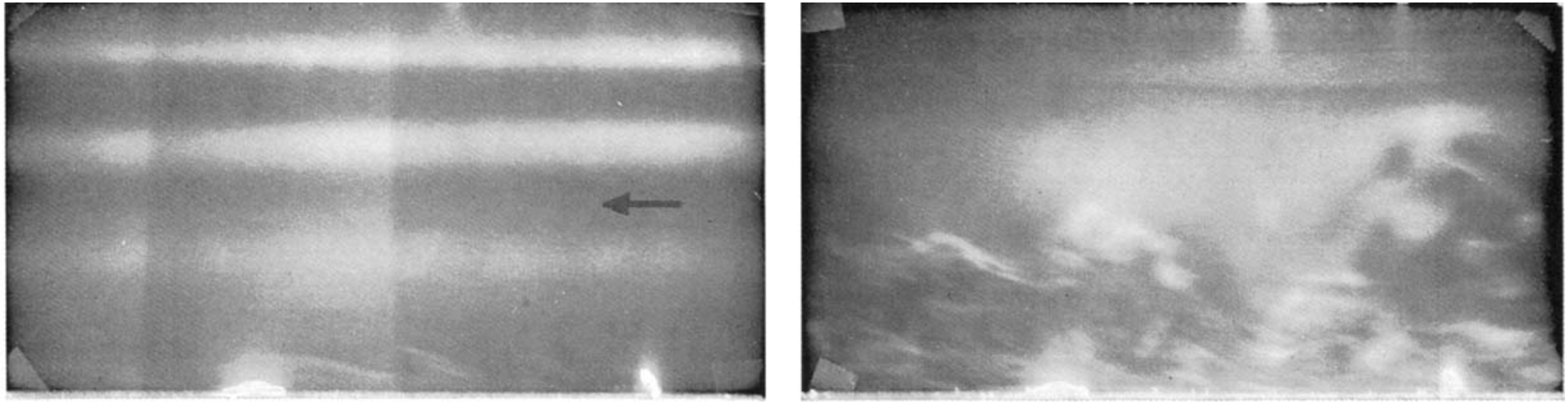
The work described here was performed at the Institute of Hydraulic Research, University of Iowa, Iowa City, under the direction of Dr Enzo O. Macagno. Financial assistance was provided by the U.S. Army Research Office (Durham) and the National Science Foundation.

REFERENCES

CHARLES, M. H. & LILLELEHT, L. U. 1965 *J. Fluid Mech.* **22**, 217.

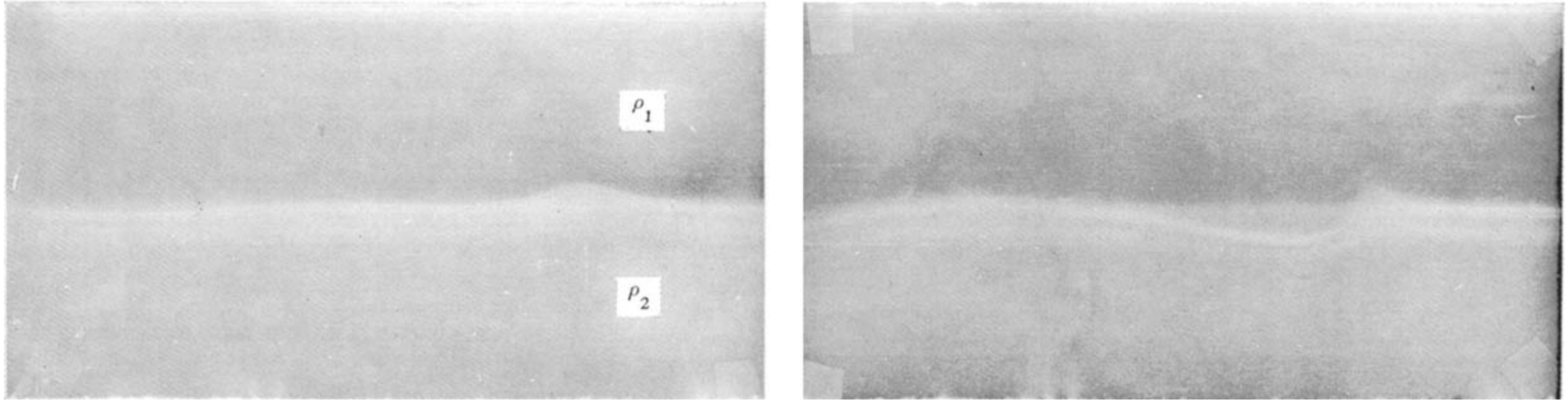
MACAGNO, E. O & HINWOOD, J. B. 1964 *VIII^e Journ. De l'Hydraulique, Soc. Hydro. Tech. France.*

MACAGNO, E. O. & HINWOOD, J. B. 1965 Final Report of Inst. of Hydraulic Research, Iowa City, to U.S. Army Research Office (Durham) on Interfacial Effects in Stratified Fluids.



(*a*) Stable flow

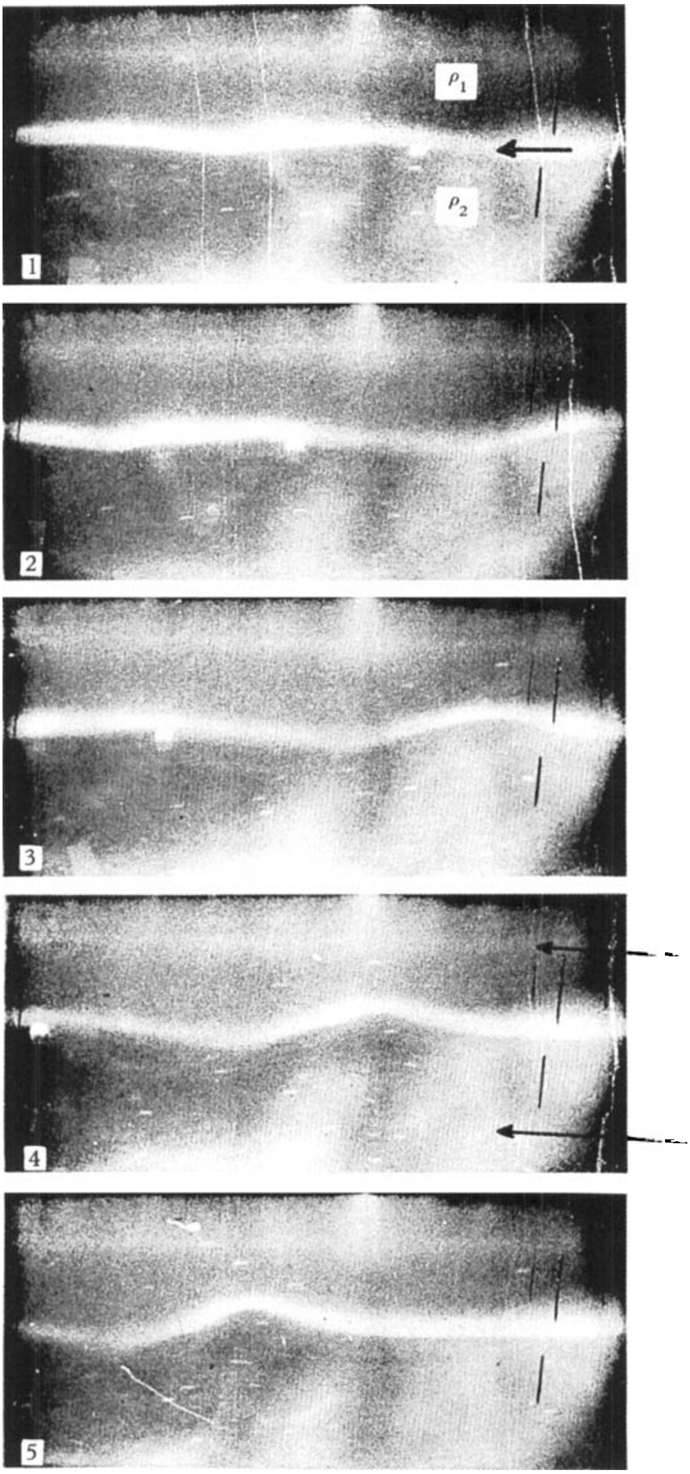
(*b*) Unstratified flow: transition



(*c*) Stratified flow: transition

(*d*) Stratified flow: onset of instability

FIGURE 6. Transition of stratified and unstratified flows.



time between photos 0·02 sec.

$x/h = 65{\cdot}4$
$F^{-1} = 0{\cdot}12$
$R = 3400$

undisturbed boundary layer shown by straight bands

bands obscured by fogging of film

FIGURE 7. Growth of unstable wave.